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FORCED OSCILLATIONS OF A FLUID IN A CYLINDRICAL TANK OSCILLATING IN A CARRIED ACCELERATION FIELD - A CORRECTION

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FOREWORD

The solution for the perfect fluid forces on a tank oscillating in a carried acceleration field as given in Reference (1) are in error through the use of an incomplete free surface boundary condition. The present report presents a corrected solution.

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SUMMARY

The correct boundary condition at the free surface of a fluid in a tank oscillating in a carried acceleration field (as in a freely falling missile) is derived and then applied to the potential function solution of Reference (1). Results are written in the form of transfer functions giving the transverse force and moment about the tank base for arbitrary planar motions of the tank.

A mechanical analogy of fixed mass plus pendulous mass is given which duplicates exactly the forces due to the first (fundamental) fluid mode.

INTRODUCTION

A cylindrical tank, partially filled with a liquid, is considered to translate and rotate in an arbitrary manner along and about transverse axes through its base.

The object of this report is to present the correct perfect fluid hydrodynamic solution for tank forces and moments for the case in which the acceleration field is carried with (rotates with) the tank.

NOMENCLATURE

A_1, B_2, C_1, D_1	- dimensionless parameters defined in equations (7) and (8)	
F	- force in x direction	- pounds
J	- Bessel function of first kind	
K_n	- tank parameter, $\xi_n h/a$	
M	- total fluid mass	- slugs
$\mathcal{M}$	- hydrodynamic moment on tank	- lb.ft.
p	- hydrodynamic pressure	- psf
a	- tank radius	- feet
h	- depth of fluid	- feet
q	- total fluid particle velocity $= \sqrt{u^2+v^2+w^2}$	- fps
r	- radial coordinate	- feet
s	- La Place variable	- sec.$^{-1}$
t	- time	- sec.
u, v, w	- fluid velocities in coordinate directions r, ϕ, z respectively	- fps
x, y, z	- cartesian coordinates	
r, ϕ, z	- polar coordinates	
α_T	- acceleration in Z (axial) direction	- fps^2
β_n	- coefficient of fluid mode	- feet^2/sec.
ξ_n	- root of $J_1' = 0$	
θ	- rotation of tank about its base	
ρ	- fluid density	- slugs/ft^3
ω_n	- fluid mode natural frequency	- sec.$^{-1}$

NOMENCLATURE (CONTINUED)

$\bar{\Omega}_n$	- defined attenuating frequency	- sec.$^{-1}$
ϕ	- fluid velocity potential : $\vec{q} = -Grad\ \phi$	- feet^2/sec.
Γ_p	- angle of pendulum with tank axis	
Γ_F	- analogous fluid "angle" defined by equation (9)	

ANALYSIS

Reference (1) presents a solution to the equations of motion and boundary conditions for a perfect fluid in a cylindrical tank undergoing small arbitrary translations and rotations (see Fig. 1). Unfortunately, the free surface boundary condition employed therein for the case of the carried acceleration field is incomplete.

To the writer's knowledge, the correct boundary condition was given first by Gleghorn in Reference 2, although the solution derived therefrom was in error.

To derive the correct free surface boundary condition for the carried field case, reference is made to Fig. 2. For a static tipping, it is seen that the total pressure at any point below the free surface is given by (θ small)

$$P = \rho a_T \left[h - z - r\theta \cos\phi \right]$$

Hence, for the case of fluid sloshing, the addition of the disturbance pressure term gives

$$\frac{P}{\rho} = \frac{\partial \Phi}{\partial t} + a_T \left[h - z - r\theta \cos\phi \right] \qquad (1)$$

where Φ is the velocity potential defined by $\bar{q} = -\mathrm{Grad}\,\Phi$. Now at the free surface ($z = h$) one must always have

$$\frac{DP}{Dt} = \frac{\partial P}{\partial t} + \frac{\partial P}{\partial r}\frac{dr}{dt} + \frac{\partial P}{\partial \phi}\frac{d\phi}{dt} + \frac{\partial P}{\partial z}\frac{dz}{dt} = 0$$

Substituting Eq. (1), letting $dz/dt = w = -\partial\Phi/\partial z$ and discarding higher order terms, one obtains the linearized boundary condition as

$$\frac{\partial^2 \Phi}{\partial t^2} + a_T \frac{\partial \Phi}{\partial z} = a_T\, r\dot{\theta} \cos\phi \qquad (2)$$

at $z = h$.

Eq. (2) is the complete free surface boundary condition for the carried acceleration field case. If the potential function of Reference (1) is now treated with Eq. (2) as the free surface boundary condition, the following results are obtained (details omitted since they parallel those of Reference 1):

<u>Velocity Potential</u>

$$\Phi = -s(\theta z + x)\, r \cos\phi + 2as \sum \frac{s^2 x + \theta h(s^2 + \bar{\Omega}_n^2)}{(s^2+\omega_n^2)(\xi_n^2-1)J_1(\xi_n)} J_1\!\left(\frac{\xi_n r}{a}\right) \cos\phi \frac{\cosh\frac{\xi_n z}{a}}{\cosh\frac{\xi_n h}{a}}$$

$$+ s\theta \sum \frac{a}{\xi_n} \frac{4a\, J_1\!\left(\frac{\xi_n r}{a}\right)}{(\xi_n^2-1)J_1(\xi_n)} \cos\phi \frac{\sinh\frac{\xi_n(z-h)}{a}}{\cosh\frac{\xi_n h}{a}} \qquad (3)$$

where $\omega_n^2 = \dfrac{a_T \xi_n}{a} \tanh \dfrac{\xi_n h}{a}$

$$\bar{\Omega}_n^2 = \frac{2a_T}{h}\left(\frac{\cosh\frac{\xi_n h}{a} - 1}{\cosh\frac{\xi_n h}{a}}\right)$$

and the ξ_n are the roots of $J_1' = 0$.

<u>Disturbance Pressure</u>

$$p_d = \rho \frac{\partial \Phi}{\partial t} = \rho s \Phi \qquad (4)$$

Integration of the pressures over the tank walls and base leads to the transverse force and moment about the tank base.

$$F = -s^2 x M - s^2\theta \left\{ M\frac{h}{2} + 4Mh \sum \frac{\cosh K_n - 1}{K_n^2(\xi_n^2-1)\cosh K_n} \right\}$$

$$+ 2M \sum \frac{s^4 x + s^2 h\theta(s^2+\bar{\Omega}_n^2)}{K_n(\xi_n^2-1)(s^2+\omega_n^2)} \tanh K_n \qquad (5)$$

$$\mathcal{M} = -s^2 x Mh\left\{\frac{1}{2} + \frac{a^2}{4h^2}\right\} - s^2\theta Mh^2\left\{\frac{1}{3} + 4\sum \frac{2\sinh K_n - K_n}{K_n^3(\xi_n^2-1)\cosh K_n}\right\}$$

$$+ s^2\, 2Mh \sum \frac{\left[s^2 x + h\theta(s^2+\bar{\Omega}_n^2)\right]}{K_n^2(\xi_n^2-1)\cosh K_n} \frac{(2 + K_n\sinh K_n - \cosh K_n)}{(s^2+\omega_n^2)} \qquad (6)$$

where $M = \pi \rho a^2 h$

$$K_n = \frac{\xi_n h}{a}$$

As in Reference (1) these results are specialized for effects of the fundamental fluid sloshing mode alone (subscript "1" in the summations).

$$F = -s^2 x M - s^2 \theta h M\left(\frac{1}{2} + C_1\right) + s^2 M A_1 \frac{s^2 x + \theta h\left(s^2 + \bar{\Omega}^2\right)}{s^2 + \omega^2} \qquad (7)$$

$$\mathcal{M} = -s^2 x Mh\left\{\frac{1}{2} + \frac{a^2}{4h^2}\right\} - s^2 \theta Mh^2\left\{\frac{1}{3} + D_1\right\} + s^2 Mh B_2 \frac{s^2 x + \theta h\left(s^2 + \bar{\Omega}^2\right)}{s^2 + \omega^2} \qquad (8)$$

Here $A_1 = \dfrac{2}{K_1}\dfrac{\tanh K_1}{(\xi_1^2 - 1)}$

$$B_2 = 2\frac{1}{K_1^2}\frac{2 + K_1 \sinh K_1 - \cosh K_1}{(\xi_1^2 - 1)\cosh K_1}$$

$$C_1 = \frac{4}{K_1^2}\frac{\cosh K_1 - 1}{(\xi_1^2 - 1)\cosh K_1}$$

$$D_1 = 4\frac{1}{K_1^3}\frac{2\sinh K_1 - K_1}{(\xi_1^2 - 1)\cosh K_1}$$

$$\xi_1 = 1.84$$

Note that here and following, the two frequency parameters written without subscript are understood to be for the fundamental sloshing mode.

<u>Mechanical Analogy</u>

Equations (7) and (8) are put into the form from which a mechanical analogy is derived by means of the substitution

$$\Gamma_F - \frac{h\bar{\Omega}^2}{\alpha_T}\theta = -\frac{1}{L_p}\frac{s^2 x + h\theta\left(s^2 + \bar{\Omega}^2\right)}{s^2 + \omega^2} \qquad (9)$$

where $L_p = \alpha_T / \omega^2$

After some rearranging one obtains*

$$
\left.
\begin{aligned}
F &= -s^2 \chi M (1 - A_1) - s^2 \theta h M \left(\frac{1}{2} + \frac{a^2}{4h^2} - B_2 \right) + M A_1 \alpha_T \Gamma_F \\
\mathfrak{M} &= -s^2 \chi M h \left(\frac{1}{2} + \frac{a^2}{4h^2} - B_2 \right) - s^2 \theta M h^2 \left(\frac{1}{3} + D_1 - B_2 \right) + M h B_2 \alpha_T \Gamma_F \\
(s^2 + \omega^2)\, \Gamma_F &= -\frac{1}{L_p} \left[s^2 \chi + s^2 \theta h \left(1 - \frac{\bar{\Omega}^2}{\omega^2} \right) \right]
\end{aligned}
\right\} \quad (10)
$$

The equations of motion for the mechanical system of Fig. 3 are identical in form with the above, being

$$
\left.
\begin{aligned}
F &= -s^2 \chi M_0 - s^2 \theta h_0 M_0 + M_1 \alpha_T \Gamma_F \\
\mathfrak{M} &= -s^2 \chi h_0 M_0 - s^2 \theta (M_0 h_0^2 + I_0) + M_1 h_1 \alpha_T \Gamma_F \\
(s^2 + \omega^2)\, \Gamma_F &= -\frac{1}{L_p} \left[s^2 \chi + s^2 \theta (h_1 - L_p) \right]
\end{aligned}
\right\} \quad (11)
$$

A term-by-term comparison between equations (10) and (11) yields the equivalences between the hydrodynamic and mechanical system given in Table 1.

* In writing these equations use has been made of the identity

$$\frac{1}{2} + C_1 - A_1 \equiv \frac{1}{2} + \frac{a^2}{4h^2} - B_2 .$$

TABLE 1

ANALOGOUS MECHANICAL SYSTEM PARAMETERS
CARRIED ACCELERATION FIELD

Mechanical	Hydrodynamic
M_0	$M(1-A_1)$
h_0	$h\left(\frac{1}{2}+\frac{a^2}{4h^2}-B_2\right)\Big/(1-A_1)$
M_1	MA_1
$Mh_0^2+I_0$	$Mh^2\left(\frac{1}{3}+D_1-B_2\right)$
h_1	$h\,B_2/A_1$
L_p	α_T/ω^2
* h_1-L_p	$h\left(1-\bar{\Omega}^2/\omega^2\right)$

*This relationship is already satisfied by a preceding equality, i.e., we can show that

$$B_2/A_1 \equiv 1-\frac{\bar{\Omega}^2}{\omega^2}+\frac{L_p}{h}$$

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DISCUSSION

The mechanical system of Fig. 3, whose parameters are given in Table 1, duplicates exactly the forces and moments on the tank and the fluid dynamic response for the fundamental fluid mode. Thus, the analogy should prove convenient for mechanization in missile stability investigations.

Analytically, the present result differs from that of Reference (1) only in the definition of the term $\bar{\Omega}$. This change results in reducing the excitation of fluid due to tank rotation over that found for the carried acceleration field case of Reference (1). The effect, in terms of the mechanical analogy, is to lower by a distance L_p the station at which the pendulum exciting acceleration acts, thereby making the locations and magnitudes of both the exciting acceleration and the feedback forces identical in the fixed and carried acceleration field cases.

The mechanical analogy found herein is therefore identical with that found (correctly) in Reference (1) for the fixed acceleration field case. In addition to being mathematically correct, this identical correspondence is asthetically pleasing, inasmuch as it appears undesirable to require a shift in mechanical analogy upon the change over from fixed to carried acceleration fields (as was the case in Reference (1)).

A further superiority of the present result lies in the fact that the equations of forces and motion, as written for the mechanical analogy in the carried acceleration field, are now correct from the point of view of pure mechanics. The same was not true of the carried acceleration field mechanical analogy of Reference (1), whose equations of motion were made to duplicate the (incorrect) hydrodynamic solution and were therefore themselves incorrect.

CONCLUSION

The correct perfect fluid solution for forces and moments on a cylindrical tank of fluid undergoing small transverse displacements and rotations in a carried acceleration field ~~has been~~ derived. A mechanical analogy has been found which duplicates exactly the forces, moments and dynamic response of the fundamental fluid sloshing mode.

REFERENCES

(1) Schmitt, A. F., "Forced Oscillations of a Fluid in a Cylindrical Tank Undergoing Both Translation and Rotation", Convair Astronautics Report ZU-7-069, October 16, 1956.

(2) Gleghorn, G. J., "Motion of Fluid in a Cylindrical Tank", Ramo-Wooldridge Corporation Memorandum, September 27, 1955.

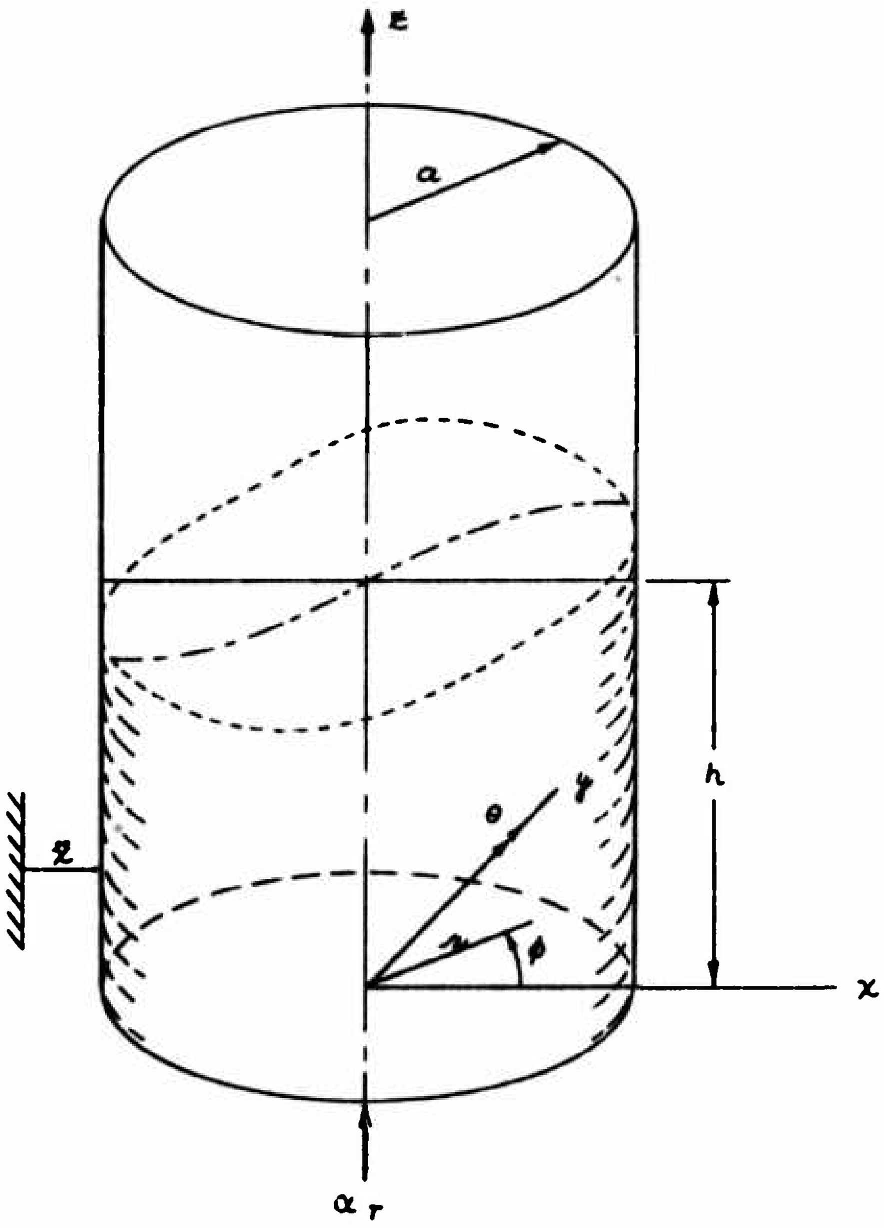

FIGURE 1

PROBLEM COORDINATES & DIMENSIONS

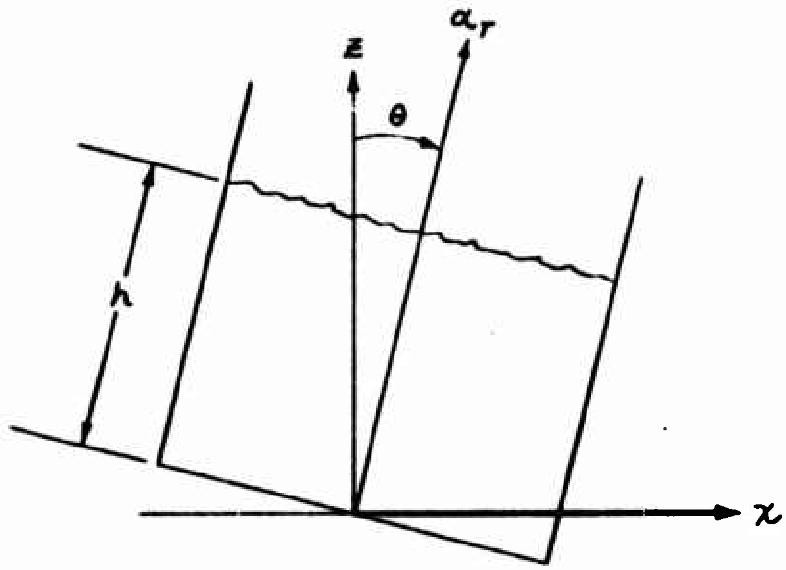

FIGURE 2

TANK ROTATING IN A CARRIED ACCELERATION FIELD

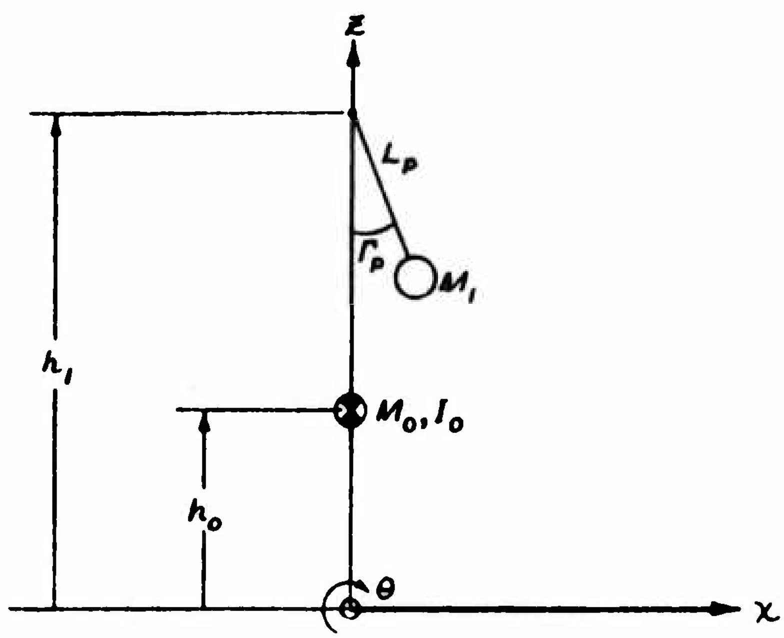

FIGURE 3

ANALOGOUS MECHANICAL SYSTEM